*Forum for Operation Oceanography 2025*

# Using old statistics and new data to predict and characterise ocean currents at scales smaller than 100 km

*Matt Rayson, University of Western Australia*

**ITRH | TIDE**

## Co-authors and Funds

Rick de Kreij, UWA

Andrew Zulberti, UWA

Will Edge, UWA

Lachlan Astfalck, UWA

Nicole Jones, UWA

Andrew Zammit-Mangion, University of Wollongong

Jeff Hansen, UWA

Paul Branson, CSIRO

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With:

ARC Linkage Project with RPS Australia

ARC Discovery Project

UWA Shell Chair

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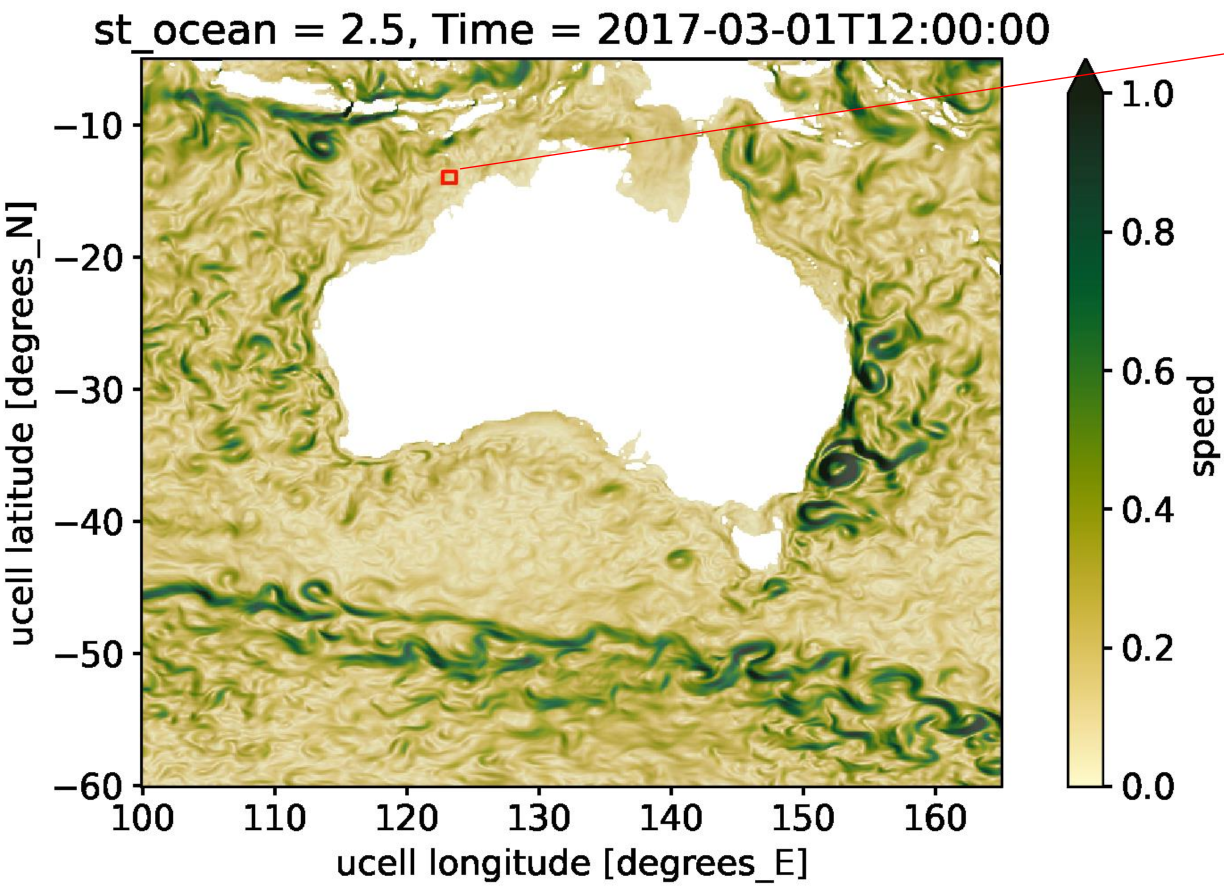
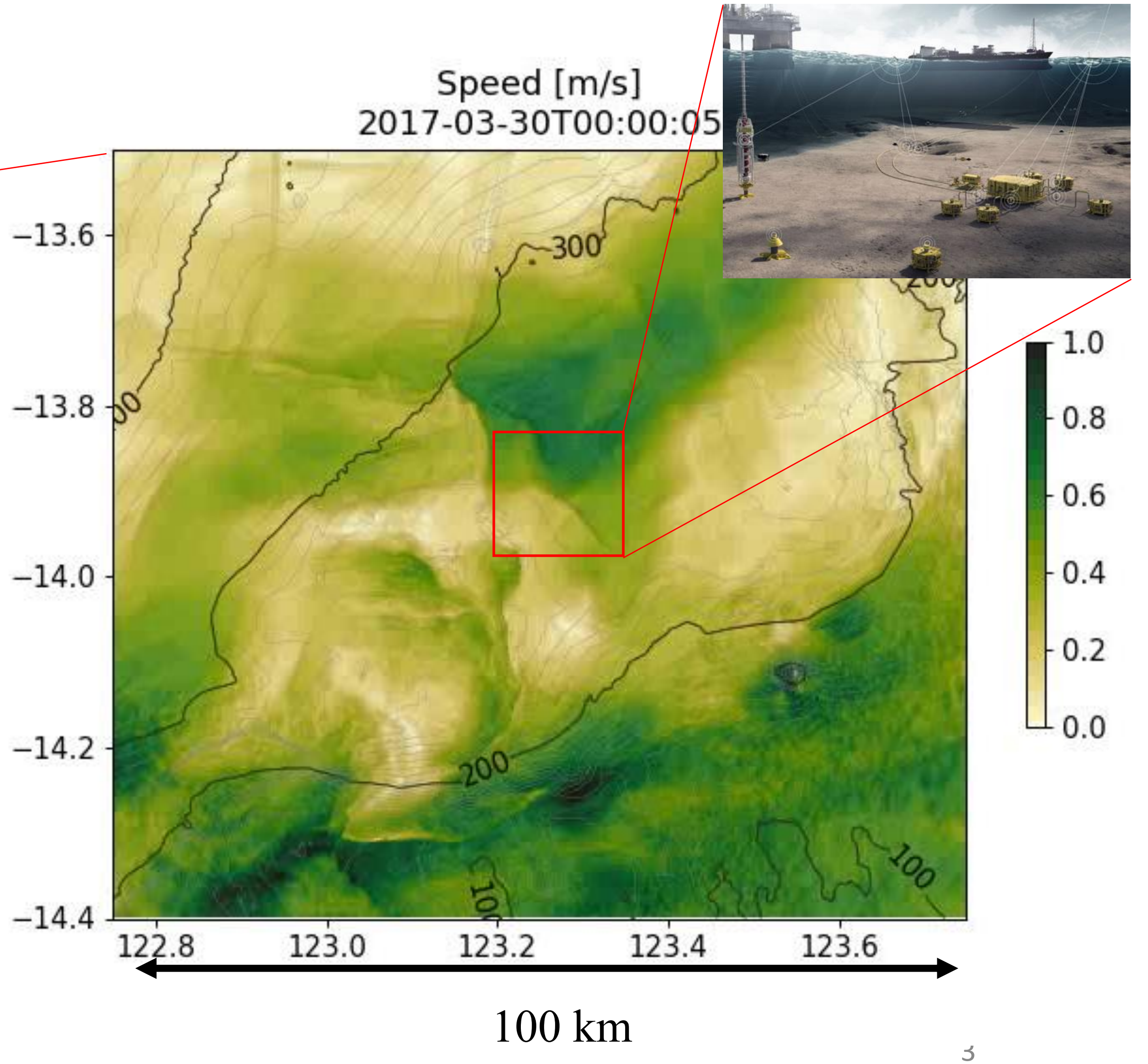
# Surface currents at different spatial scales



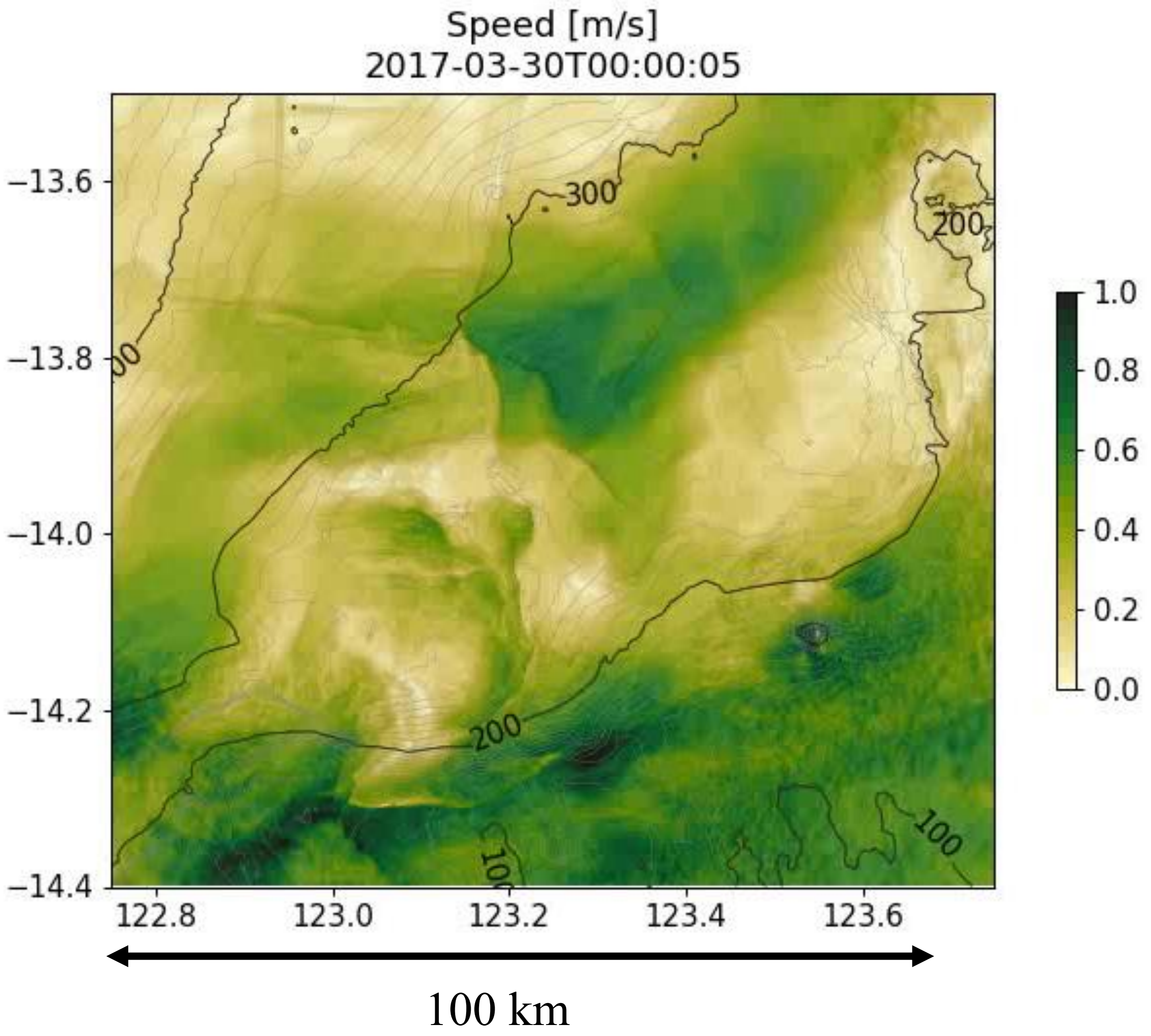



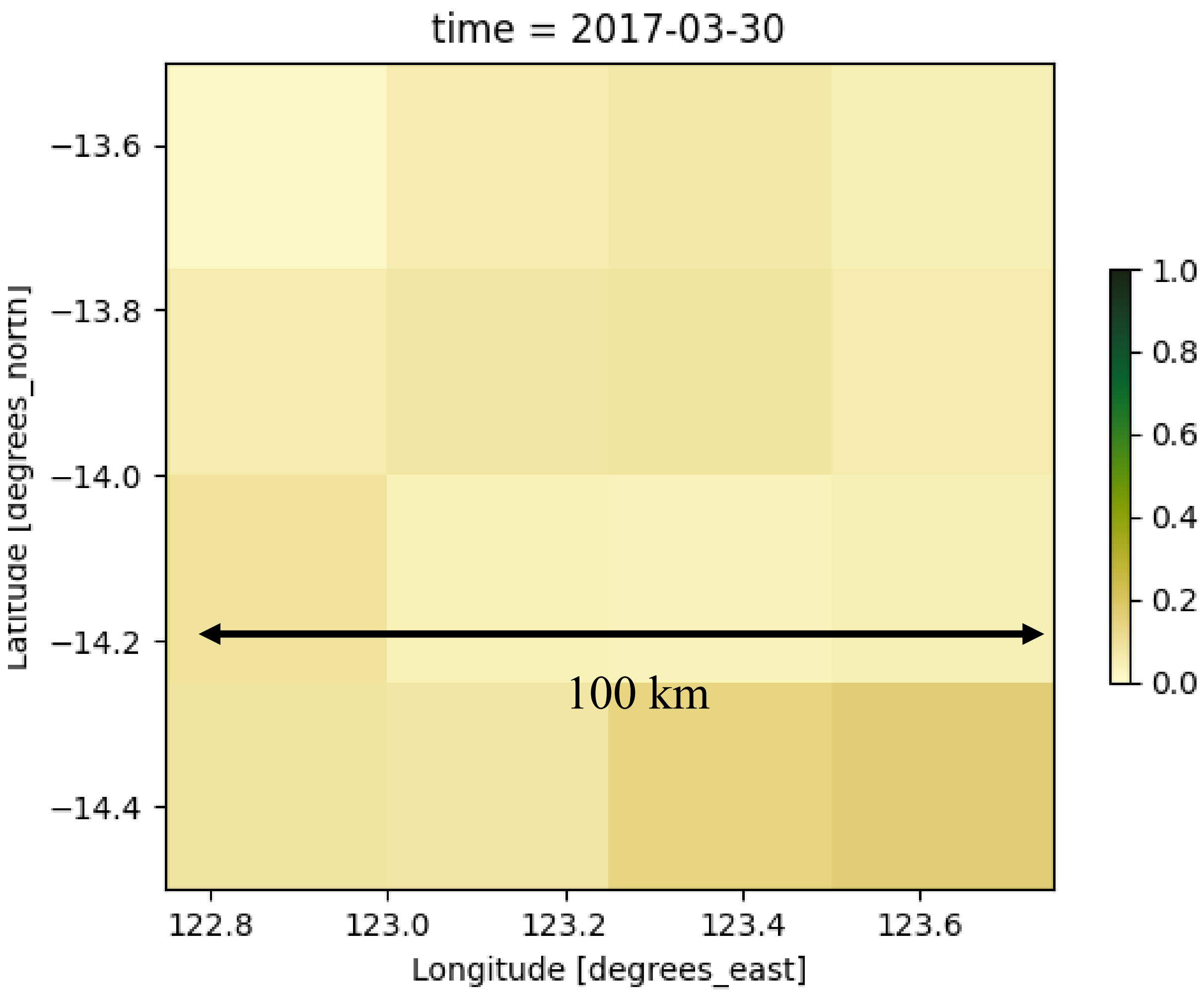

**Bluelink Reanalysis (v2020), 10 km resolution**

**SUNTANS Nonhydrostatic Shelf-Scale (0.125 km resolution)**

# Spatial scale of operational surface current products

Copernicus GlobCurrent 0.25x0.25 degrees (c.f. IMOS OceanCurrent GSLA product)



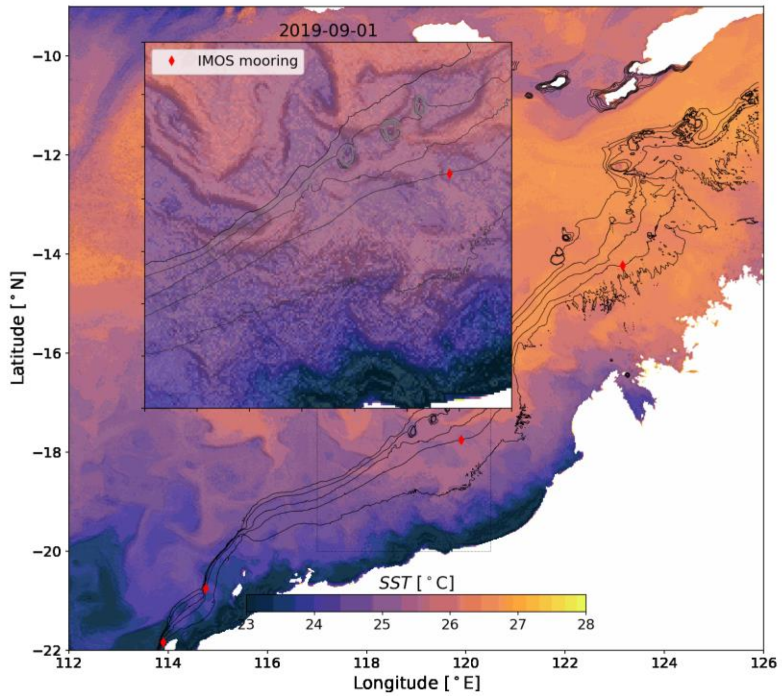



# Spatial scales of new remote sensing data

**Himawari-8/9**

Sea surface temperature

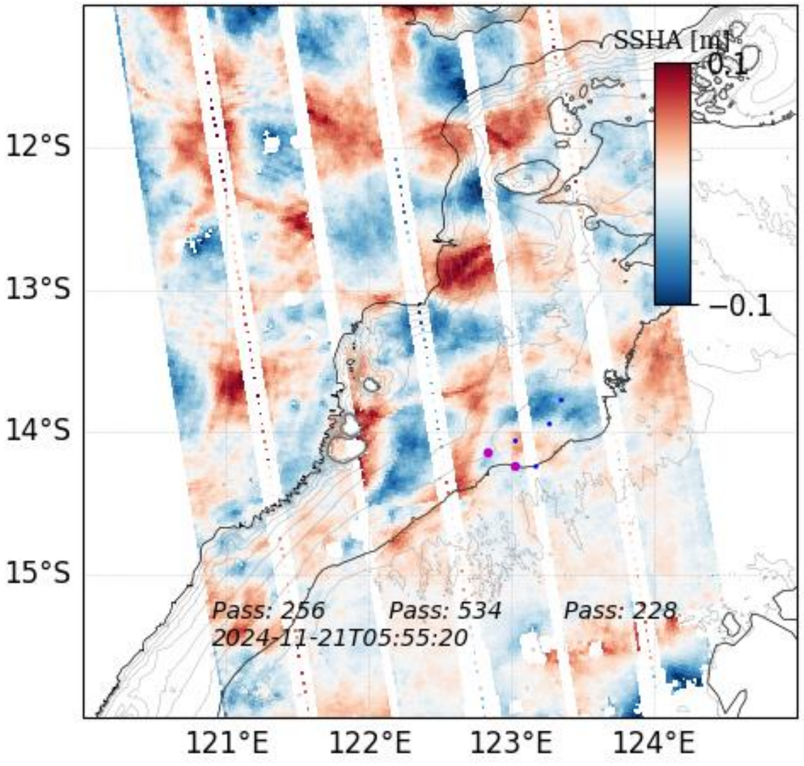
dx = 2 km, dt = 1 hr




**SWOT**

Sea surface height

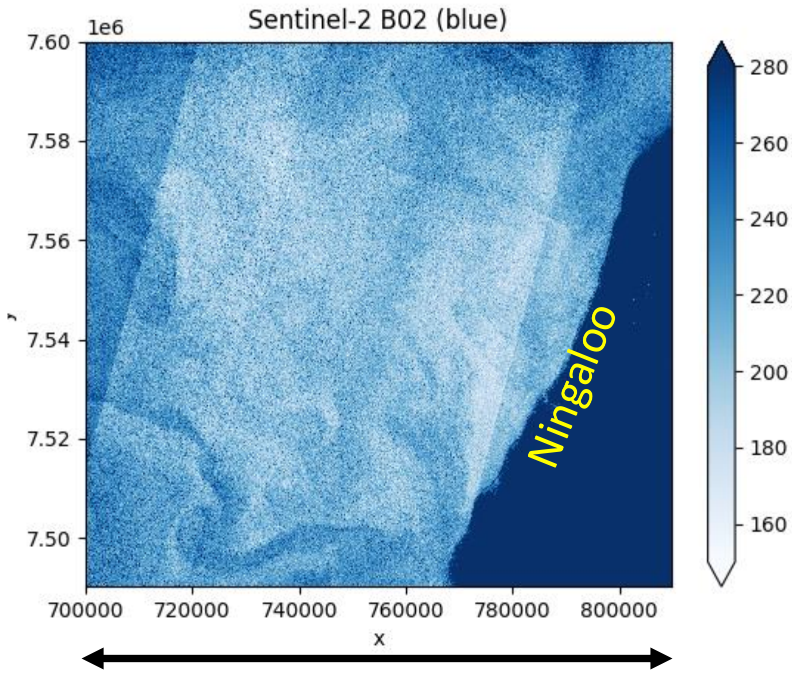
dx = 2 km, dt = 21 days




**Sentinel-2a/b/c**

Optical (wave phase)

dx = 10 m, dt = 1 s (every 3-5 days)



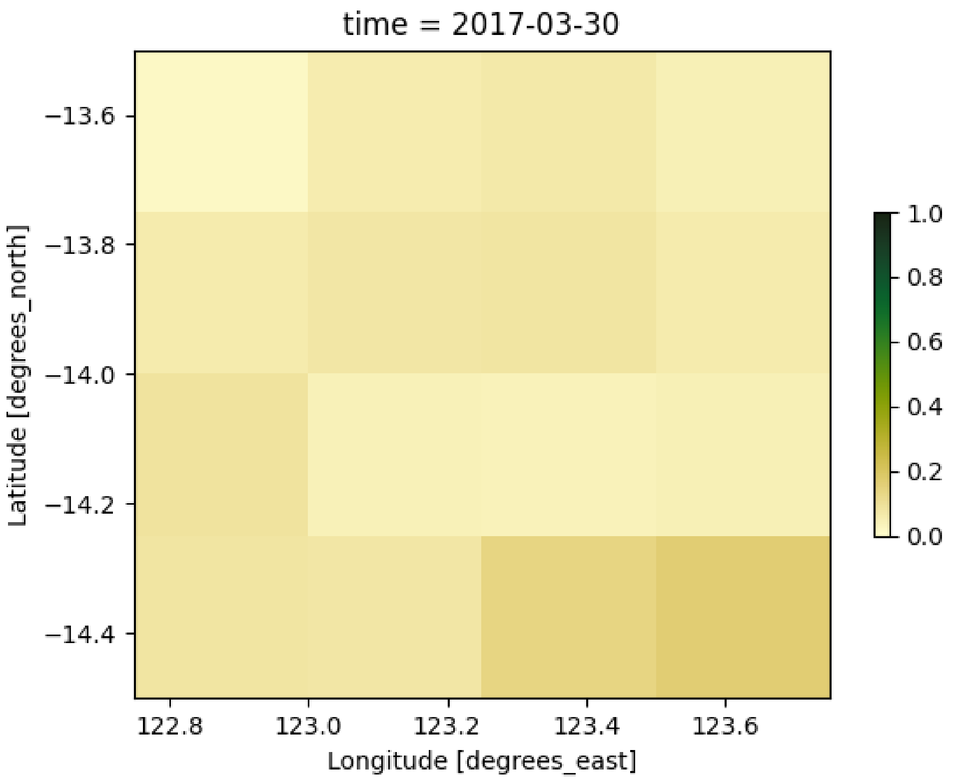

100 km

# Goal

Extract quantitative predictions of surface currents at scales smaller than 100 km using new generation, high-resolution satellite observations

Or

Fill in the gaps here



# Methodology

*Linear* model linking observations to the physics (i.e., surface currents)

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \varepsilon$$

where:

- **y** is a vector of observations, typically not surface currents
- **x** is a vector of surface currents (*u* and *v* velocity)
- **H** is a “physics” operator e.g., equations that link observed variables in y to unobserved velocities
- $\varepsilon$ is the error

→ The goal is to invert the equations to find **x**

## Methodology - solution

One approach is to solve x using *least-squares*

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \varepsilon$$

$$\mathbf{x} = (\mathbf{H}^T\mathbf{H})^{-1}\mathbf{H}^T\mathbf{y}$$

Matlab code:

```
% solve y = Hx
x = H/y
```

Python code:

```
# solve y = Hx
import numpy as np
x = np.linalg.lstsq(H,y)
```

→Doesn't work! Too many unknowns
→Need to add some constraints on **x** (the velocity)

## Methodology – Bayesian solution

An optimal approach is to solve **x** by assuming it is a Gaussian distribution i.e., the velocity is a probability distribution and not a single estimate

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \varepsilon$$

Priors

$$\mathbf{x} \sim GP(0, \mathbf{B})$$

$$\varepsilon \sim GP(0, \mathbf{R})$$

→Priors (**B** and **R** ) are multivariate normal (Gaussian) distributions
→ The priors constrain the solution
→Choosing **B** and **R** is *really* important
→Solution naturally comes with uncertainty quantification

Solution (mean and variance)

$$\mathbb{E}[\mathbf{x}] = \mathbf{B}\mathbf{H}^T(\mathbf{H}\mathbf{B}\mathbf{H}^T + \mathbf{R})^{-1}\mathbf{y}$$

$$\mathbb{V}[\mathbf{x}] = \mathbf{B} - \mathbf{B}\mathbf{H}^T(\mathbf{H}\mathbf{B}\mathbf{H}^T + \mathbf{R})^{-1}\mathbf{H}\mathbf{B}$$

# Example 1: Currents from sea surface temperature

Geostationary Himawari-8/9 SST

Physics

$$\frac{\partial T}{\partial t} = -\frac{\partial T}{\partial x}u - \frac{\partial T}{\partial y}v + q$$

Discrete linear system

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \varepsilon$$

$$\mathbf{y} = \begin{bmatrix} \frac{\partial T_1}{\partial t} \\ \vdots \\ \frac{\partial T_s}{\partial t} \end{bmatrix}$$

$$\mathbf{H} = \begin{bmatrix} -\frac{\partial T_1}{\partial x} & \cdots & 0 & -\frac{\partial T_1}{\partial y} & \cdots & 0 & 1 & \cdots & 0 \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ 0 & \cdots & -\frac{\partial T_s}{\partial x} & 0 & \cdots & -\frac{\partial T_s}{\partial y} & 0 & \cdots & 1 \end{bmatrix}$$

$$\mathbf{x} = \begin{bmatrix} \begin{bmatrix} u_1 \\ \vdots \\ u_s \end{bmatrix} \\ \begin{bmatrix} v_1 \\ \vdots \\ v_s \end{bmatrix} \\ \begin{bmatrix} q_1 \\ \vdots \\ q_s \end{bmatrix} \end{bmatrix}$$

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# Example 1: Currents from sea surface temperature

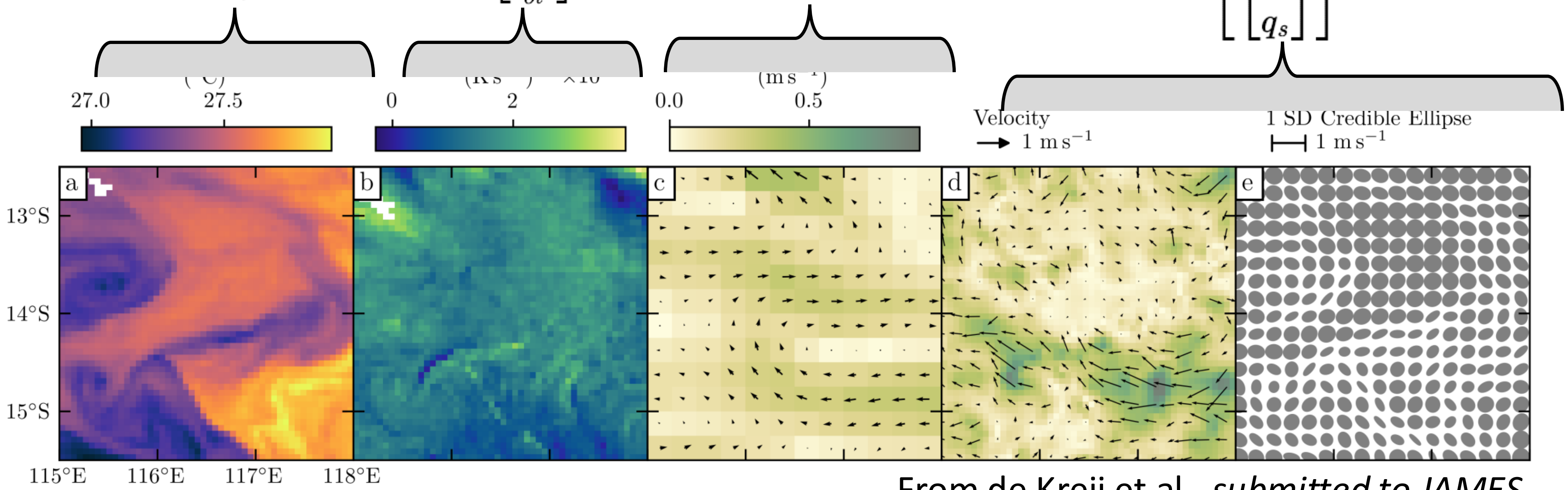
Geostationary Himawari-8/9 SST

## Discrete linear system

$$\mathbf{x} = \begin{bmatrix} \begin{bmatrix} u_1 \\ \vdots \\ u_s \end{bmatrix} \\ \begin{bmatrix} v_1 \\ \vdots \\ v_s \end{bmatrix} \\ \begin{bmatrix} q_1 \\ \vdots \\ q_s \end{bmatrix} \end{bmatrix}$$

$$\mathbf{H} = \begin{bmatrix} -\frac{\partial T_1}{\partial x} & \cdots & 0 & -\frac{\partial T_1}{\partial y} & \cdots & 0 & 1 & \cdots & 0 \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ 0 & \cdots & -\frac{\partial T_s}{\partial x} & 0 & \cdots & -\frac{\partial T_s}{\partial y} & 0 & \cdots & 1 \end{bmatrix} \quad \mathbf{y} = \begin{bmatrix} \frac{\partial T_1}{\partial t} \\ \vdots \\ \frac{\partial T_s}{\partial t} \end{bmatrix}$$

Independent data (altimetry)




From de Kreij et al., *submitted to JAMES*

# Example 2: Internal tide induced currents from sea surface height

Surface water ocean topography

## Physics

*Oscillatory currents proportional to sea surface height gradient*

## Discrete linear system

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \varepsilon$$

- **y** is a vector of sea surface height observation
- **x** is a vector of sea surface height *harmonic amplitudes*
- **H** contains cosine and sine series with different tidal frequencies

# Example 2: Internal tide induced currents from sea surface height

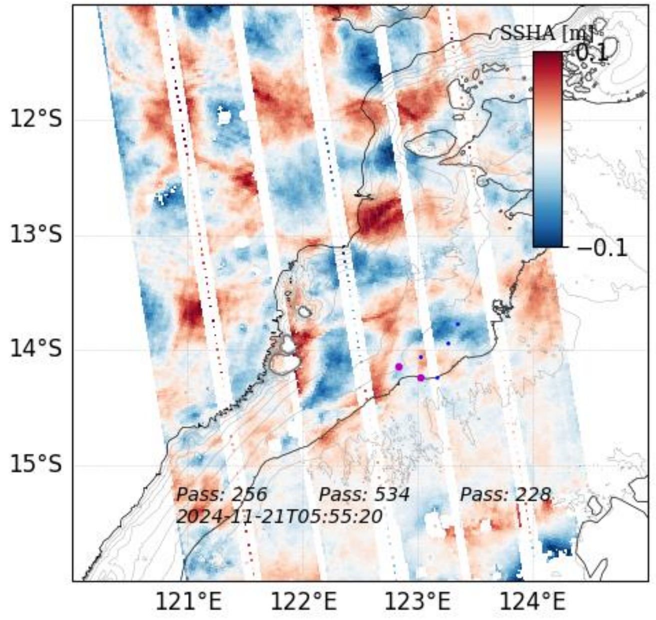
Surface water ocean topography

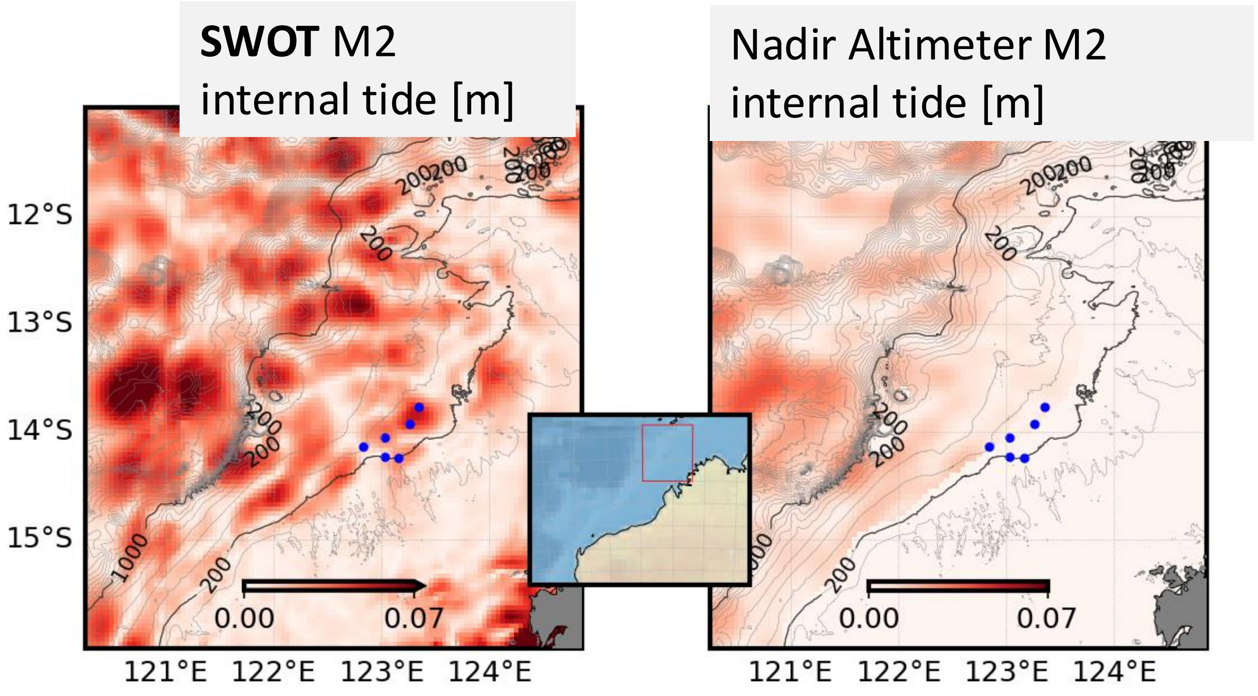
$$\mathbf{y} = \mathbf{H}\mathbf{x} + \varepsilon$$

**H** contains cosine and sine series with different tidal frequencies

**x** is a vector of sea surface height *harmonic amplitudes*

**y** sea surface height observation



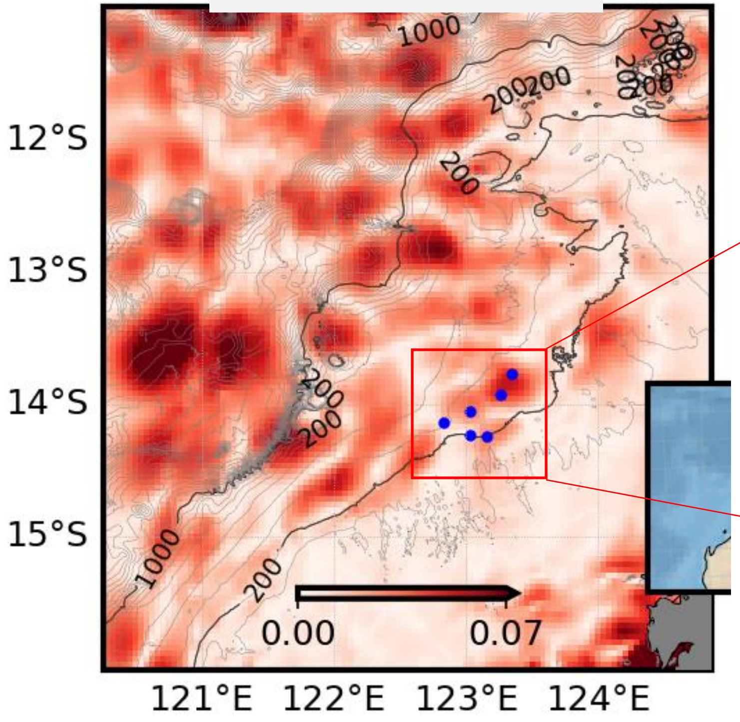



# Example 2: Internal tide induced currents from sea surface height

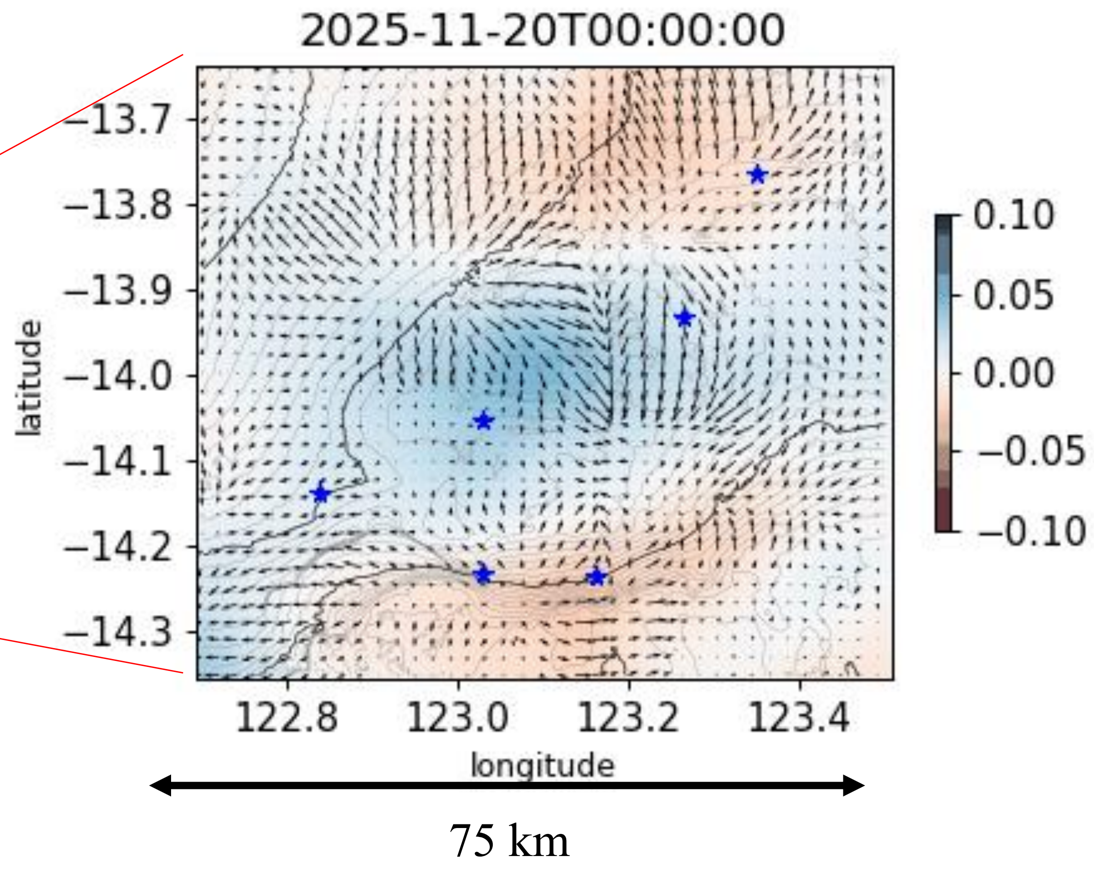
Surface water ocean topography

Surface currents proportional to sea surface height *gradient*

**SWOT** M2
internal tide [m]



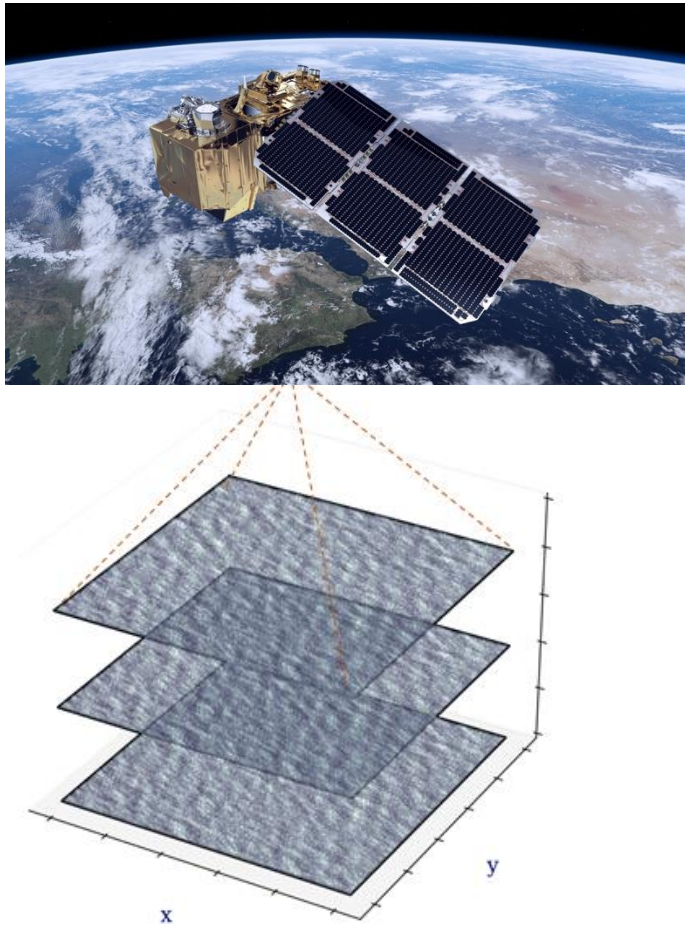

**Prediction**

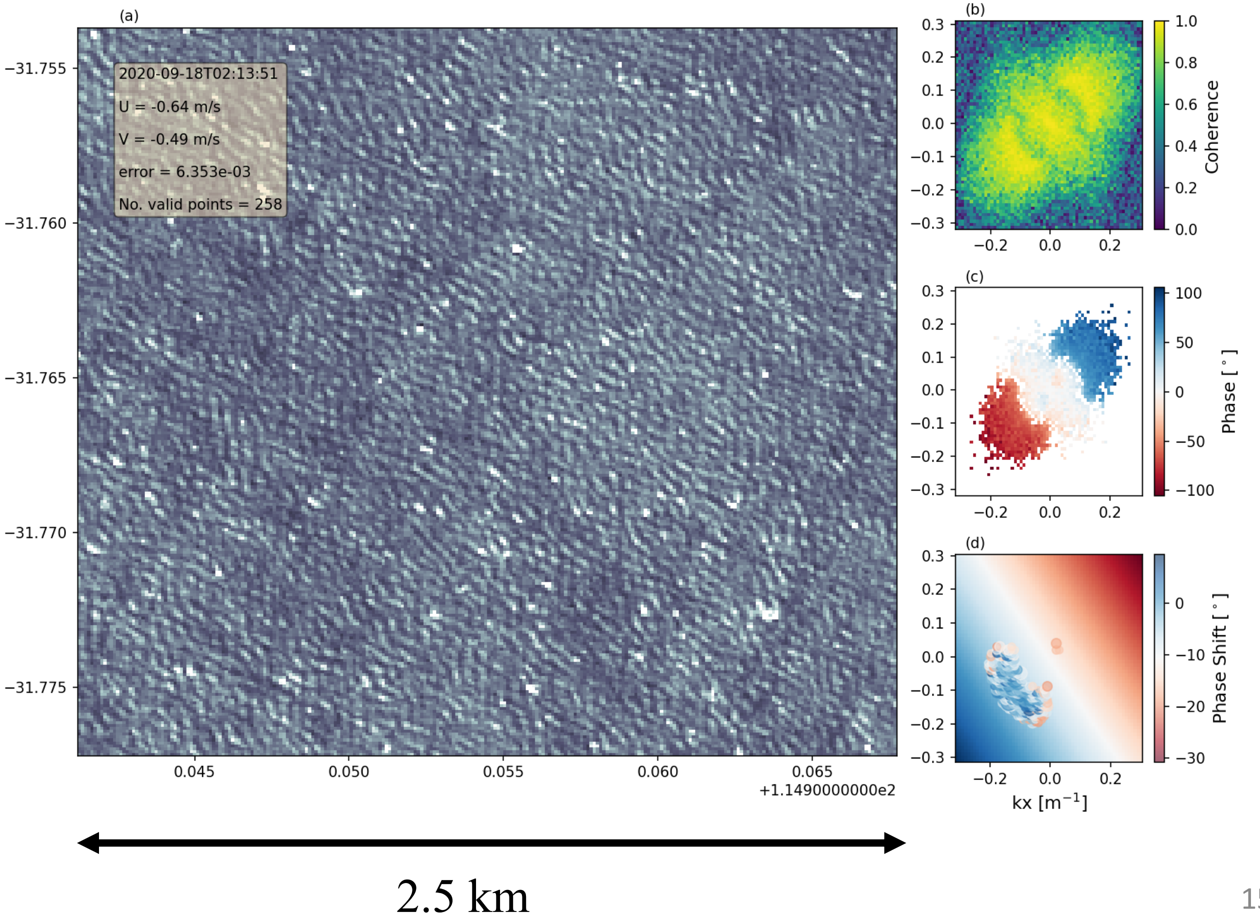


# Example 3: Currents from aerial sunglint imagery

Sentinel-2a/b/c: red, green and blue images have a ~1 second time lag

→ can detect phase propagation of surface gravity waves



# Example 3: Currents from aerial sunglint imagery

Sentinel-2a/b/c

## Physics

*Observed phase = Theoretical phase – wavenumber * time lag * velocity*

*Phase shift = Observed phase - Theoretical phase*

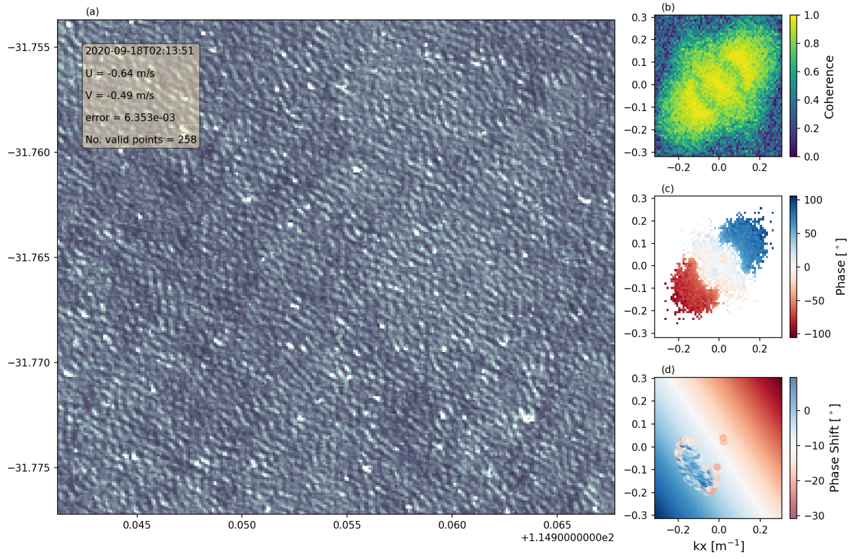
## Discrete linear system

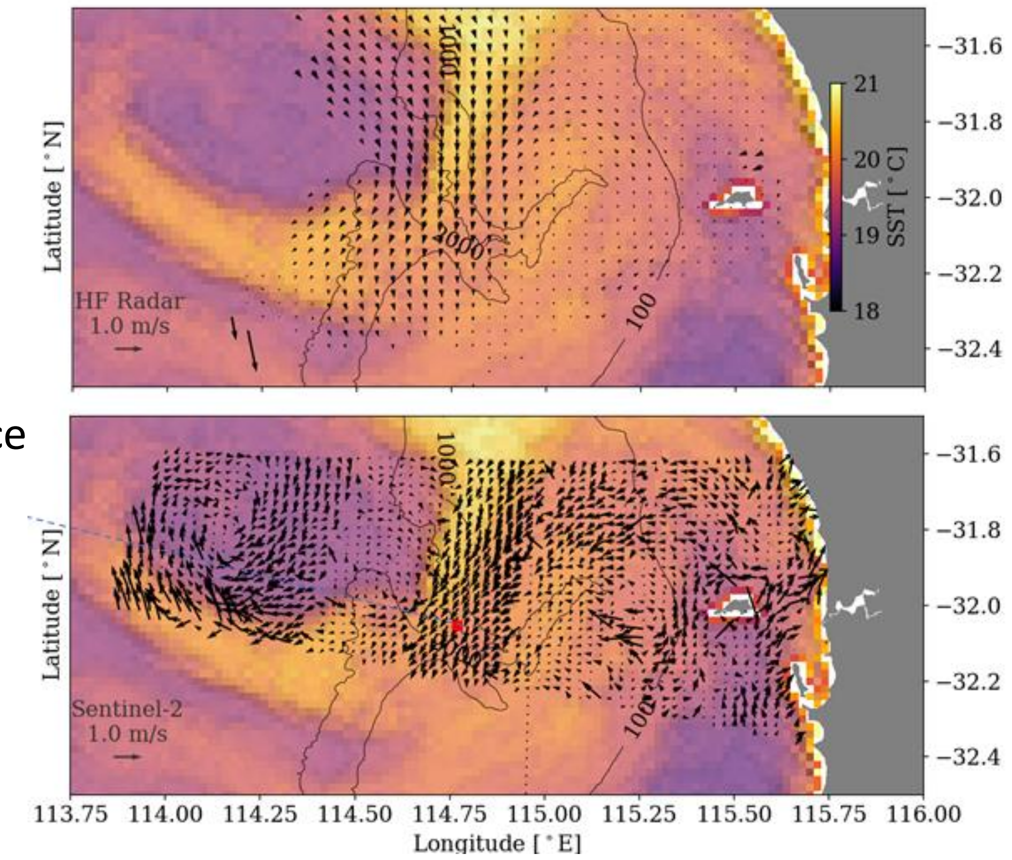
$$\mathbf{y} = \mathbf{H}\mathbf{x} + \varepsilon$$

- **y** is a vector of *wave phase shift*
- **x** is a vector of surface current vectors
- **H** contains *wavenumbers * image time lag*

# Example 3: Currents from aerial sunglint imagery

Sentinel-2a/b/c



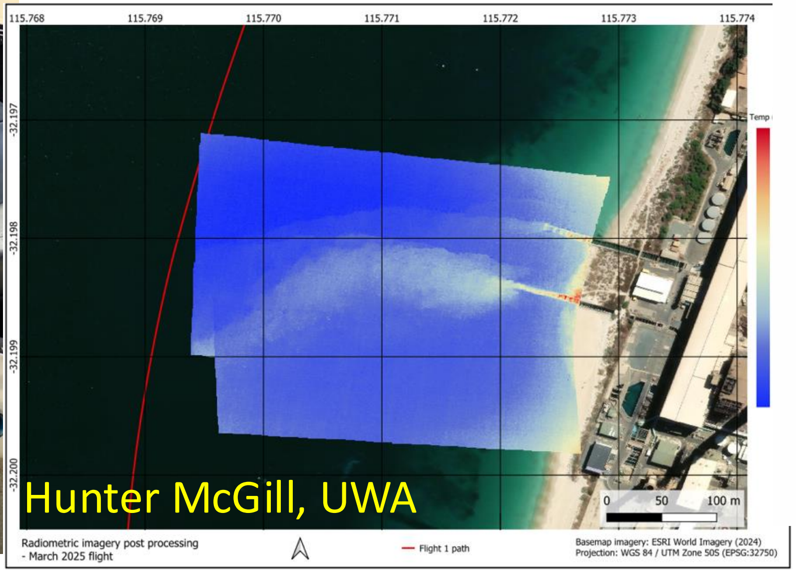

**x** is a vector of surface current vectors

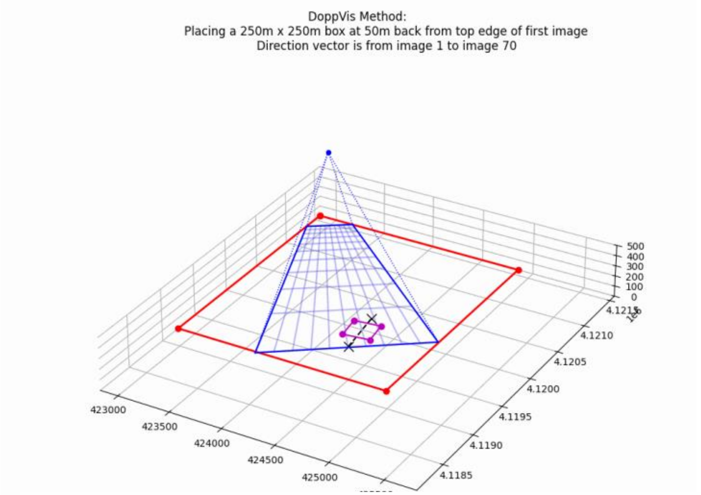
**y** is a vector of *wave phase shift*

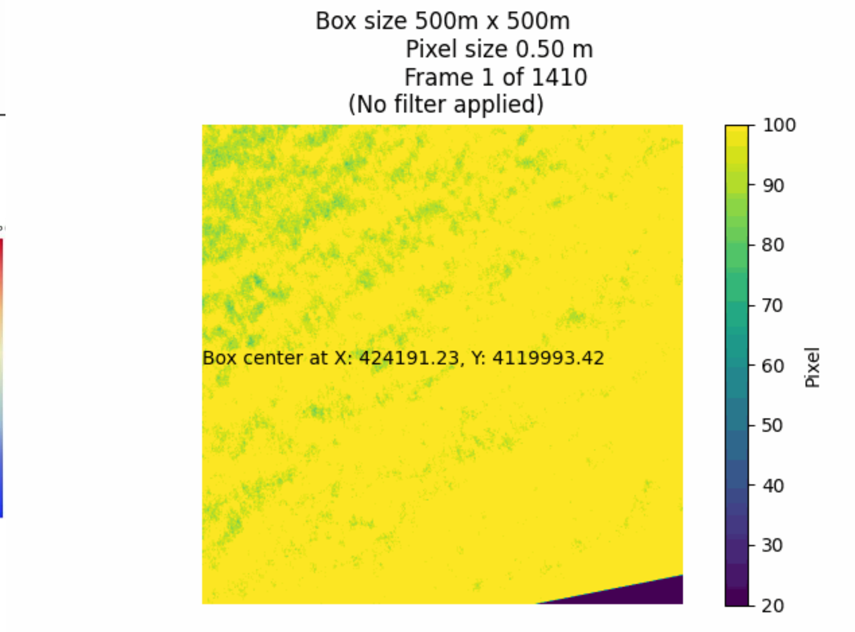
**H** contains *wavenumbers * image time lag*

# The future: bespoke optical and thermal imagery from aerial platforms

ARC Linkage Project with RPS Australia



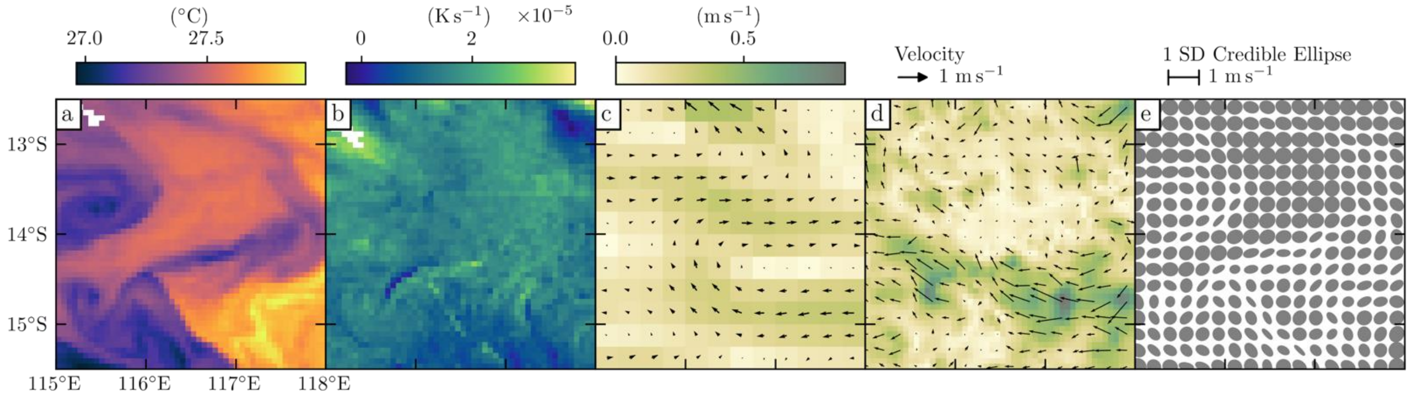



## Conclusions

Predictions of currents at scales < 100 km require:

- High quality data with appropriate sampling rates in space and time
- A physics model linking the observation variable to surface currents (simple, linear models are preferable)
- Statistical knowledge of the process, namely the spatio-temporal variability of surface currents
- *Data assimilating numerical models and/or deep learning approaches have the same requirements




Contact: matt.rayson@uwa.edu.au