Integrating physics and statistics: a Bayesian approach to predictive uncertainty of solitons

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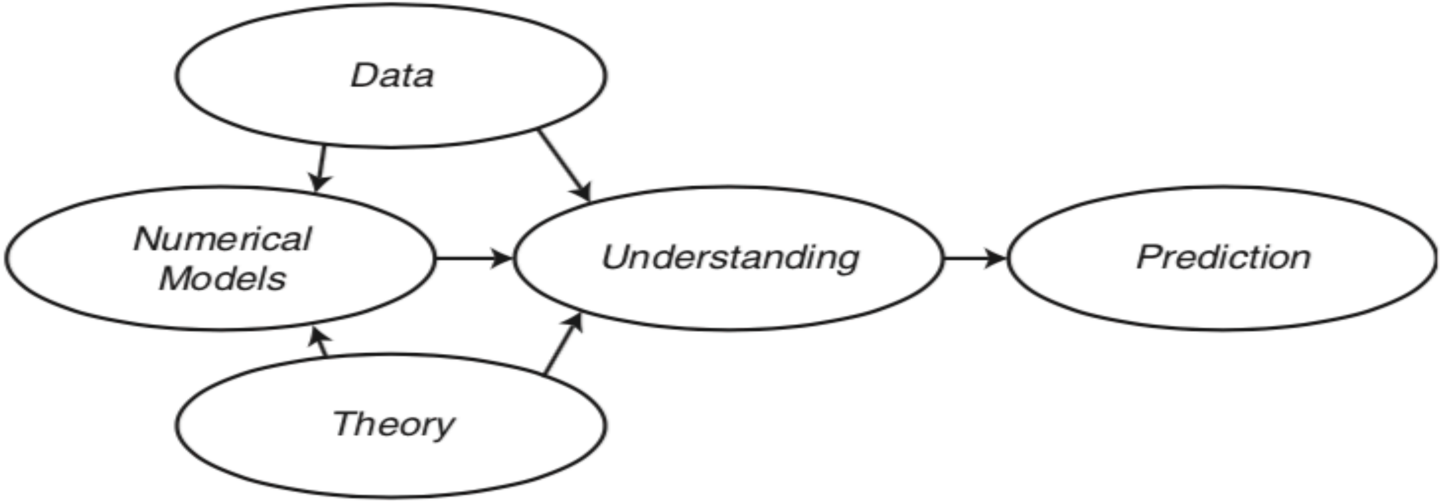
Data Science: What is it?

Our Definition: Using probability and data to reconcile real physical systems with mathematical approximations



The soliton problem

Soliton: a non-linear internal oceanic wave generated by tidal forcing over topography.

Interest for ocean engineering and oceanography because:

- Important driver of extreme currents.
- Induce large stress on offshore infrastructure.
- Influence dynamic position systems during operations.
- Drive sediment suspension

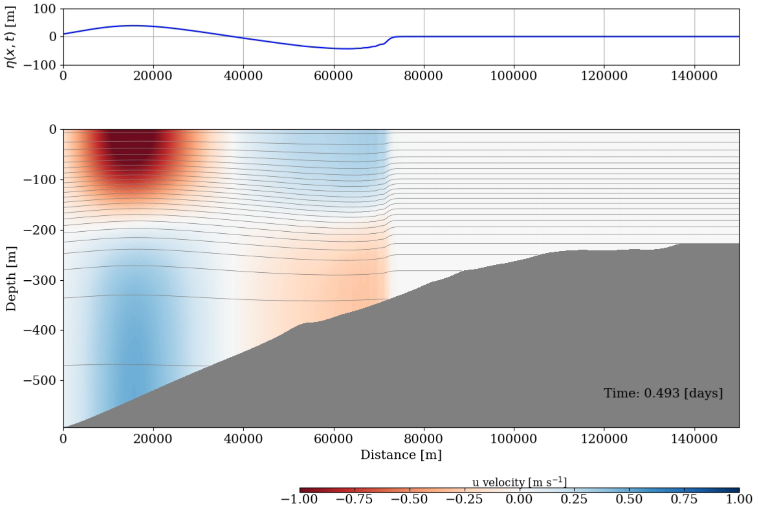
Motivating Industrial Question:

What will be a *plausible range* of maximum amplitudes induced by a given soliton.

The soliton problem

Model the amplitude, $A(x,t)$, using the variable-coefficient *KdV equation:*

$$\frac{\partial A}{\partial t} + c(x)\frac{\partial A}{\partial x} + \alpha(x)A\frac{\partial A}{\partial x} + \beta(x)\frac{\partial^3 A}{\partial x^3} + \frac{c(x)}{2Q(x)}\frac{\partial Q}{\partial x}A = 0$$



Initial Conditions (unknown):

- Density Stratification
- Initial wave amplitude

Important to acknowledge:

- Inputs are uncertain
- KdV is a simplification/approximation of reality
- Even if exact, the solution is still approximate because it is intractable.

The approximation problem

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Uncertainty and probability:

- Any form of approximation induces uncertainty
- Probability theory provides a mathematical description of uncertainty.
- Basic to all decision making under uncertainty.

"...the theory of probabilities is basically just common sense reduced to calculus; it makes one appreciate with exactness that which accurate minds feel with a sort of instinct, often without being able to account for it." Pierre Simon Laplace (1749–1827)

Bayesian Statistics

A coherent probabilistic fusion of data and scientific knowledge

new data: $p(y \mid \theta)$

prior knowledge: $p(\theta)$

$$p(\theta \mid y) \propto p(y \mid \theta)\, p(\theta)$$

posterior $\propto$ likelihood • prior

We build a model for the temporal evolution of density stratification, given data observed on the North West Shelf.

KdV Inputs: density stratification

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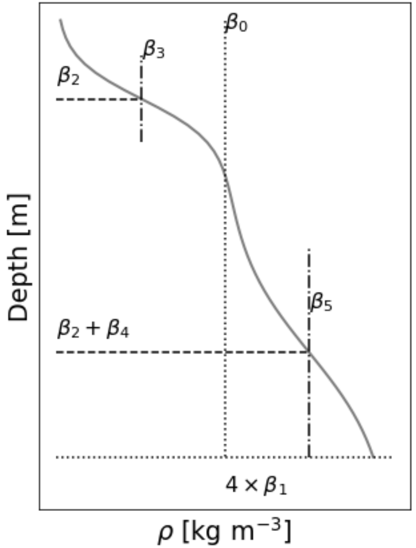
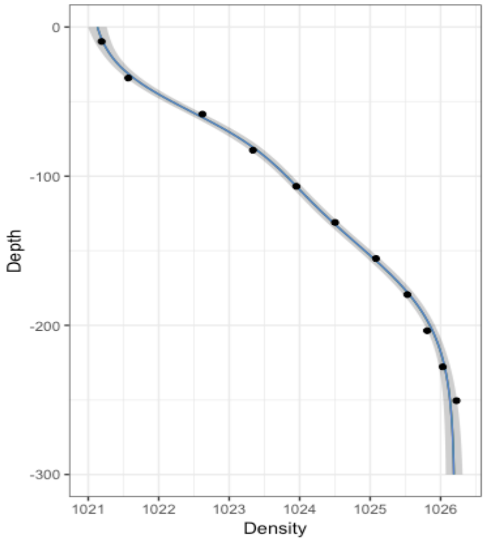
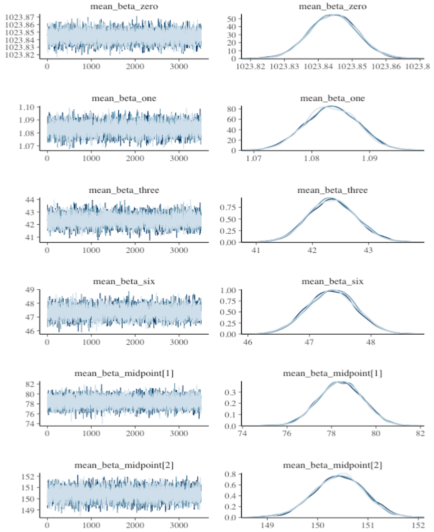
$$\rho(z) = \beta_o + \beta_1(\tanh(\frac{\beta_2 - z}{\beta_3}) + \beta_1 \tanh(\frac{\beta_4 - z}{\beta_5}))$$



2. Fit w/ uncertainty



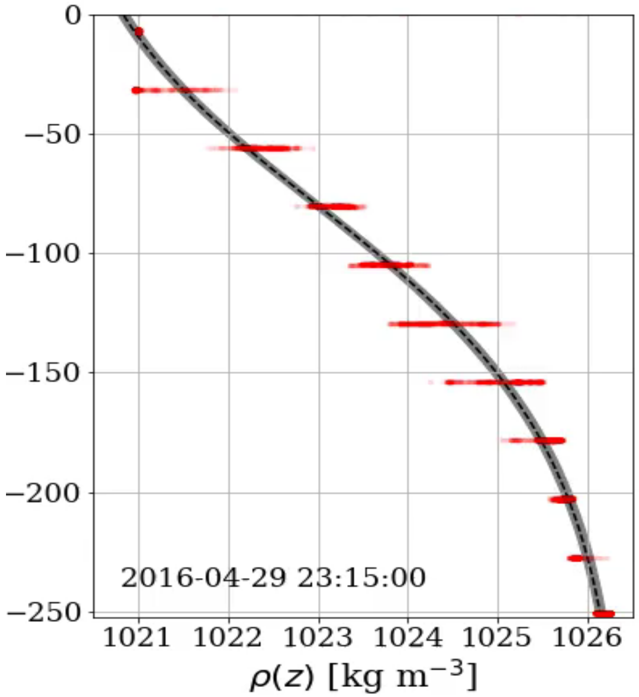
3. Parametric inference

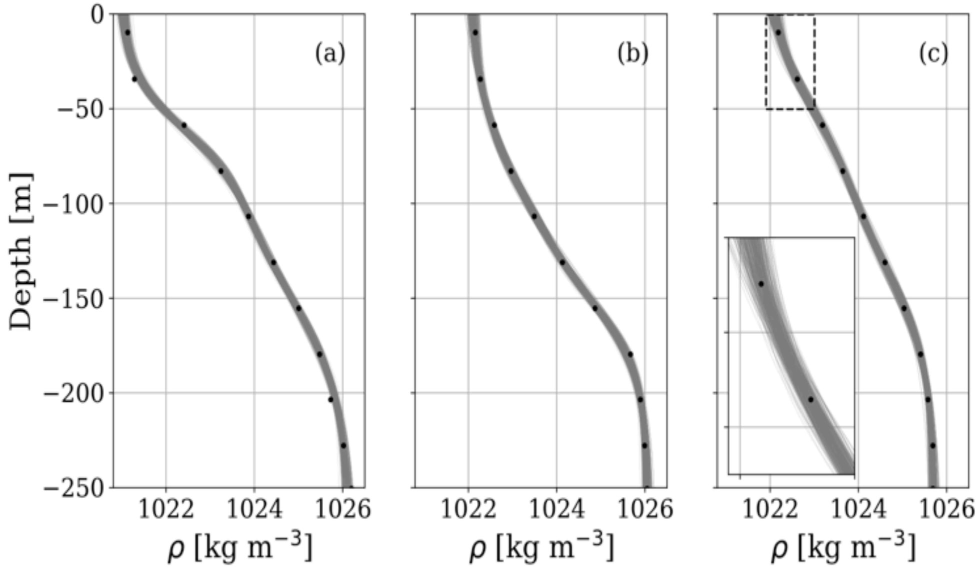


Manderson, A., Rayson, M., Cripps, E., et al. (2019)
``Uncertainty quantification of density and stratification estimates with implications for predicting ocean dynamics''.
Journal of Atmospheric and Oceanic Technology, 36(7), pp 1313--1330

KdV Inputs: density stratification

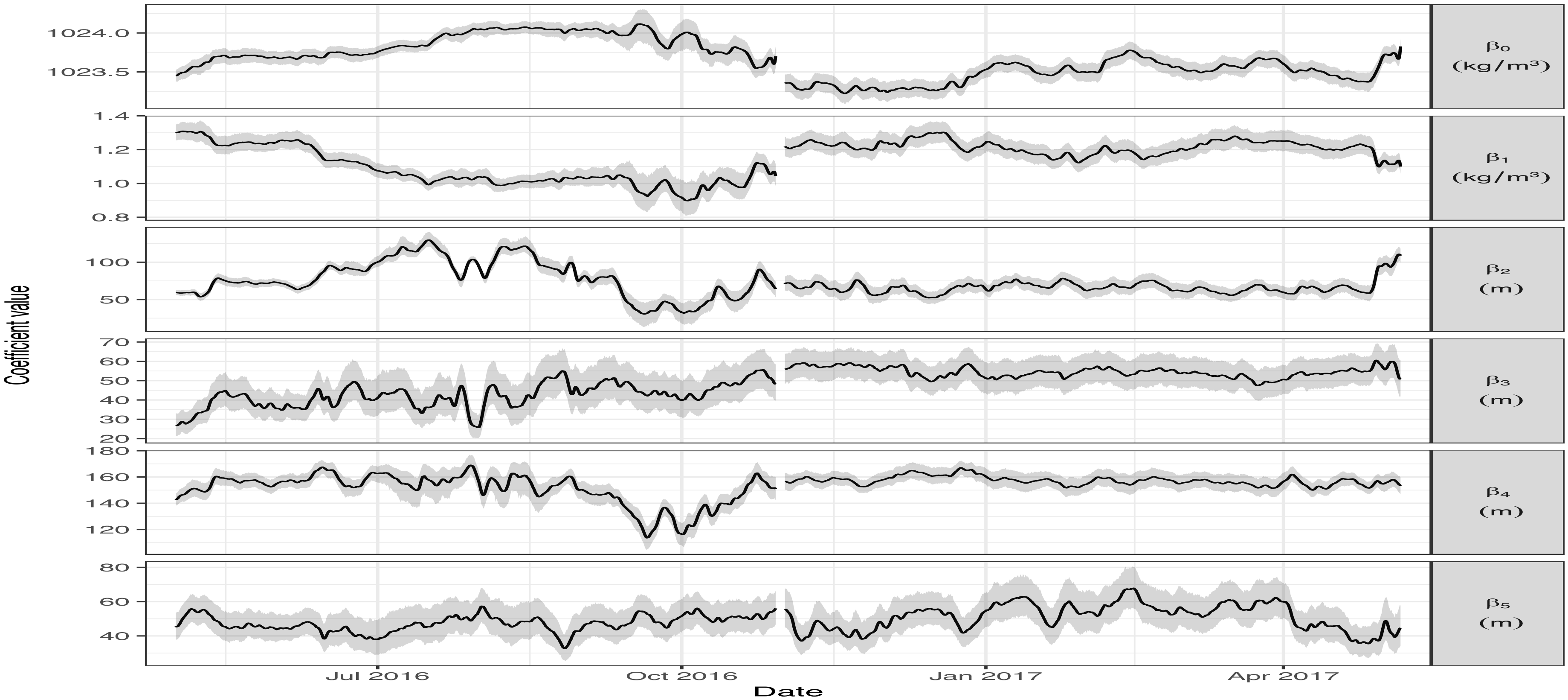
Temporal evolution: embed parametric in hierarchical model





KdV Inputs: density stratification

Temporal evolution of predictive distribution of density profile characterists



KdV Inputs: density stratification

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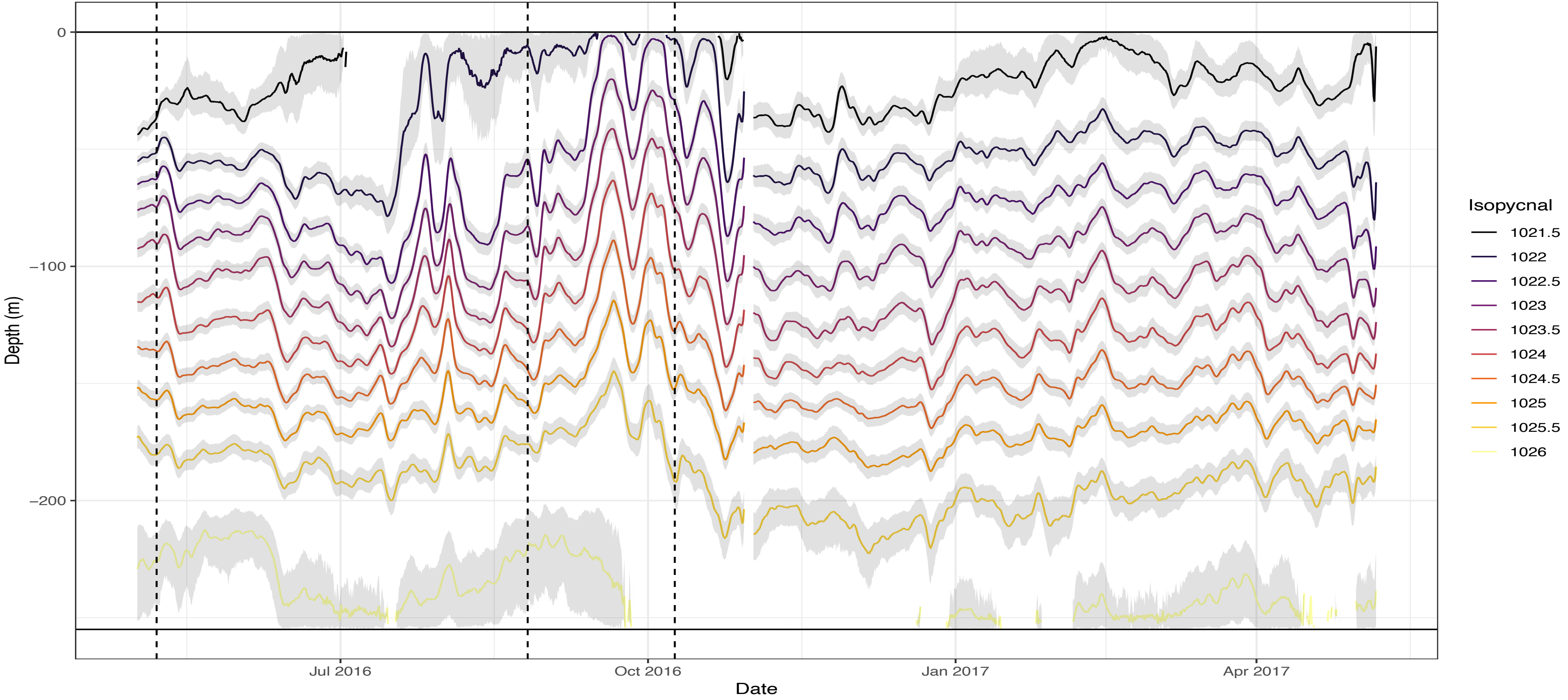
Temporal evolution of the predictive distribution of isopycnals



Uncertainty Analysis of KdV:

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Physical model: The Korteweg-de Vries equation for continuously stratified fluids:

$$\frac{\partial A}{\partial t} + c(x)\frac{\partial A}{\partial x} + \alpha(x)A\frac{\partial A}{\partial x} + \beta(x)\frac{\partial^3 A}{\partial x^3} + \frac{c}{2Q}\frac{\partial Q}{\partial x}A = 0$$

Uncertain Inputs:

- $\alpha(x)$ and $\beta(x)$ require density profile, $\rho_t(z)$, z = depth vector
- Initial condition $A(x,0)$ requires initial wave amplitude, a_0

Statistical models for input uncertainty: $p(\rho_t(z))$ and $p(a_{0t})$

e.g.,

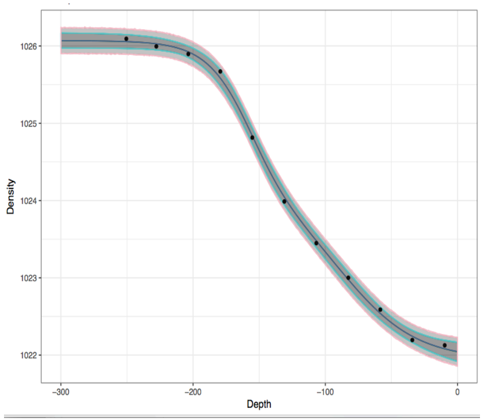
- $y_t = \rho_t + \epsilon_t,$

Combining the physics, statistics and computing for industrial impact

$p(A_{mt}|Data_t, KdV)$: the predictive distribution of A_{mt}, integrating out inputs

KdV uncertainty quantification

Predictive distribution of maximum amplitude at a given time.



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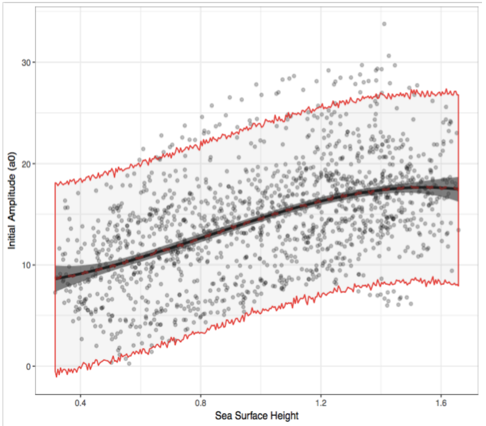

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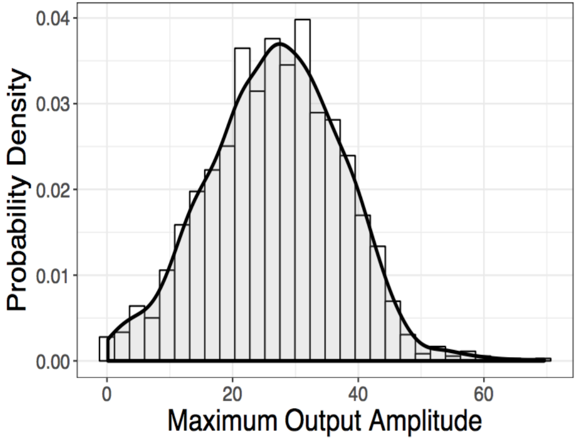

Computationally demanding: Parallel, distributed and cloud computing

Software development: Necessary for industrial uptake

The people

To achieve the previous slides needs input from a wide range of expertise:
Oceanographers, statisticians, computer engineers, engineers.

Manderson

Rayson

Barlow

Girolami

Gosling

Hodkiewicz

Ivey

Jones

Current Extensions and the bigger picture

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Bayesian Analysis of Computer Code Output

Denote by $y = f(x : \theta)$ as the simulated output of a computer model that relies on inputs x and tuning parameters, θ.

- Uncertainty Analysis: propagate $p(x)$ through $f(x : \theta)$
- Inversion: given observations, z, identify optimal values of θ via its posterior distribution: $p(\theta | x, y, z)$
- Numeric Solver Uncertainty Different solvers/grids etc, can yield different approximate evaluations of the model. Probability can turn the problem into one of statistical inference.
- Sensitivity Analysis: determines which inputs are most influential on simulator output.